Effects of Tangent Support Stiffness on Sags at High Temperature

I. The Problem

The sags in individual spans within a ruling span section are normally calculated, for temperatures and loadings different from those at the time of clipping in, under the assumption that tangent supports swing freely, with negligible longitudinal restraining force from the supports. Thus, all spans within the section are assumed to have the same tension as the calculated ruling span tension under all loading conditions. Actually, since insulator strings are not infinitely long, there must be some opposing force when temperature or loading causes longitudinal insulator offsets, and this force appears as a difference in horizontal tensions between the spans on either side of the support. The various spans do not share the same tension then, and this affects the predicted sags, an effect not normally taken into account.

II. Method of Attack

The problem can be analyzed by considering what happens when, for example, temperature and/or conductor loading is changed, breaking the process into two steps. In the first step, the conductor is considered to held by supports that are completely free to move longitudinally, tantamount to leaving the conductor in sheaves. The temperature and/or loading is changed, the new ruling span tension is calculated, and, importantly, the longitudinal displacements of the supports are determined.

In the second step, the insulator strings are attached to the longitudinally displaced suspension clamps and allowed to pull them back toward their proper positions. Because of the weight loading of the strings, they act like springs in the face of longitudinal displacement. The conductor spans also act like springs, because of their elasticity and because of the variation of tension with slack in the catenary. Thus, we have a system of springs that has been distorted by the longitudinal offsets and will adjust itself to balance out the various forces acting within and on it.

The problem can be set up as a matrix equation which takes the form,

$$\boldsymbol{M} \cdot \boldsymbol{x} = \boldsymbol{K}\boldsymbol{a} \tag{1}$$

where

$$\boldsymbol{M} = \begin{bmatrix} K_1 + k_1 + k_2 & -k_2 & & & & \\ -k_2 & K_2 + k_2 + k_3 & -k_3 & & & \\ & -k_3 & K_3 + k_3 + k_4 & -k_4 & & \\ & & -k_4 & - & & \\ & & & & - & -k_6 \\ & & & & -k_6 & K_6 + k_6 + k_7 \end{bmatrix}$$

$\boldsymbol{a}$ is the vector of displacements from the first step, and $\boldsymbol{x}$ is the vector of final support point longitudinal displacements, after the insulators have been connected and picked up their load. $\boldsymbol{K}$ is the row vector of support point stiffnesses, K_1, K_2, etc. These stiffnesses may be represented by wS_w/l, where w is conductor weight per unit length, S_w is the weight span for the support in question, and l is insulator string length. The weight of the string is ignored, or taken into account by adjusting the effective length of the insulator.

III. Step 1 - Unrestrained Longitudinal Movement of Supports

The displacements a_1, a_2, etc. at the supports due to temperature and/or load change with the conductor effectively still in sheaves are determined as follows. The change in slack in the span has five components:

- catenary "strain" due to change in sag, $\Delta\varepsilon_s$,
- elastic strain due to changed tension, $\Delta\varepsilon_c$,
- plastic strain and creep, $\Delta\varepsilon_p$,
- thermal elongation due to change in temperature, $\alpha\,\Delta t$, and
- net movement of the conductor over the sheaves into the span, ΔS.

Thus,

$$\Delta S = S\,(\Delta\varepsilon_s - \Delta\varepsilon_c - \Delta\varepsilon_p - \alpha\,\Delta t) \tag{2}$$

We want the movement ΔS.

First consider the change in catenary slack strain, ε_s. It is given by,

$$\varepsilon_s = \frac{2H}{wS}\cdot\sinh\left(\frac{wS}{2H}\right) - 1 \tag{3}$$

To a good approximation,

$$\varepsilon_s = \frac{1}{6}\left(\frac{wS}{2H}\right)^2 = \frac{1}{24}\left(\frac{wS}{H}\right)^2 \tag{4}$$

The change in ε_s can be calculated for each span by inserting into (4) the original tension and weight, H_1 and w_1, and then the calculated ruling span tension and weight, H_2 and w_2, after the changes in temperature and/or loading. This change in strain is then mulitplied by S to get the change in slack. Call the change in catenary strain $\Delta\varepsilon_s'$.

Now the elastic strain $\Delta\varepsilon_c$, the inelastic strain and thermal strain, are the same as those for the ruling span, since the tension H_2 and temperature t_2 are the same throughout the entire section before the insulators are reattached and loaded. But, for the ruling span, $\Delta S = 0$, so $\Delta\varepsilon_s = \Delta\varepsilon_c + \Delta\varepsilon_p + \alpha\,\Delta t$. $\Delta\varepsilon_c + \Delta\varepsilon_p + \alpha\,\Delta t$ can be calculated from (4) for the ruling span. Thus,

$$\Delta\varepsilon_c + \Delta\varepsilon_p + \alpha\,\Delta t = \frac{1}{24}\left(\frac{w_2^2}{H_2^2} - \frac{w_1^2}{H_1^2}\right)\cdot S_r^2 \tag{5}$$

Note that knowledge of the details of $\Delta\varepsilon_c + \Delta\varepsilon_p + \alpha\,\Delta t$ is not needed, as long as H_1, H_2, w_1 and w_2 are known for the ruling span.

Using (5) with (2) and (4), we get after manipultation,

$$\Delta S = \frac{S}{24}\cdot\left(\frac{w_2^2}{H_2^2} - \frac{w_1^2}{H_1^2}\right)(S^2 - S_r^2) \tag{6}$$

where S_r is the ruling span.

Now, ΔS is the net length of conductor that passes into the span over the sheaves at *both* ends. The support point displacements can be calculated by starting with Span 1, which is deadended at one end, and noting that its tangent support at Structure 1 must move ΔS_1. The support at Structure 2 must move that amount, plus ΔS_2. That at Structure 3 moves $\Delta S_1 + \Delta S_2 + \Delta S_3$, etc. These movements are the displacements a_1, a_2, a_3, etc., and are used in the vector $\boldsymbol{a}$ of (1). The vector on the right-hand side of (1) becomes,

$$\boldsymbol{K}\,\boldsymbol{a} = \langle K_1 a_1 \;\; K_2 a_2 \;\; K_3 a_3 \;\; \cdots \;\; K_n a_n \rangle^T \tag{7}$$

IV. Step 2 - Reconnection of Insulators to Suspension Clamps

In this step, the stiffness of the spans is considered to have two components. One is the elasticity of the conductor, and this stiffness is EA/S, where EA is the effective per unit length stiffness given by,

$$EA = E_A A_A + E_S A_S \tag{8}$$

and S is span length. Any inelastic strains are considered to have occurred in Step 1.

The other component is equal to the rate of change of tension in the span with variation in its *slack*. The slack is the difference between the arc length of the catenary and the secant span length. As noted above, the slack, expressed as a fraction of S, is,

$$\varepsilon_s = \frac{2H}{wS}\cdot\sinh\left(\frac{wS}{2H}\right) - 1 \tag{3}$$

To a good approximation,

$$\varepsilon_s = \frac{1}{6}\left(\frac{wS}{2H}\right)^2 = \frac{1}{24}\left(\frac{wS}{H}\right)^2 \tag{4}$$

so that,

$$H \approx \frac{wS}{\sqrt{24\,\varepsilon_s}} \tag{9}$$

Then,

$$k_s \;=\; -\frac{1}{S}\frac{dH}{d\varepsilon_s} \;=\; \frac{w}{\sqrt{96}\,\varepsilon_s^{\frac{3}{2}}} \;=\; 12\,\frac{H^3}{w^2 S^3} \qquad (10)$$

The last step requires some manipulation. Where there is a significant change in H, the average of k_s over that range should be used. It is,

$$\text{avg } k_s = \frac{12}{w^2 S^3}\frac{1}{H_i - H_2}\int_{H_2}^{H_i} H^3 dH = \frac{12}{w^2 S^3}\cdot\frac{H_i^4 - H_2^4}{4\,(H_i - H_2)} \qquad (11)$$

These spring constants come into play during the second step, after the conductor has in concept rolled over sheaves under the change in temperature and/or loading. Thus, the value of w to use is the loaded value.

H_2 refers to the ruling span tension after the changes in temperature and/or loading. H_i is the tension in span i after the spring system has pulled the support points back to equilibrium. This poses a difficulty, since H_i isn't known until after the calculations are carried out. Thus, iteration seems to be needed. However, ΔH is generally small enough with respect to H_2 that H_2 itself can be used in (10).

The effective spring constant for the span is,

$$k = \frac{1}{\frac{1}{k_c} + \frac{1}{k_s}} \qquad (12)$$

These give the entries for the matrix $\boldsymbol{M}$.

V. Solution for Final Equilibrium

Equation (1) is solved by taking the inverse of $\boldsymbol{M}$ and then postmultiplying it by $\boldsymbol{K}\,\boldsymbol{a}$. The result is the vector $\boldsymbol{x}$ of displacements of the insulators resulting from picking up their load and pulling the supports back toward their original positions. The *net* displacements are $\boldsymbol{a}+\boldsymbol{x}$.

The net strain that occurs in span i as a result of loading the insulators is,

$$\Delta\varepsilon_i = \frac{(a_i + x_i) - (a_{i-1} + x_{i-1})}{S_i} \qquad (13)$$

The change in tension in span i due to the displacements a is

$$\Delta H_i = k_i\,(a_i - a_{i-1}) \qquad (14)$$

These tension changes can be added to H_2 to obtain the final tensions in the individual spans.

VI. Effect of Inclined Spans

Inclined spans change the vertical loads at supports from what is calculated for level spans: conductor weight times the sum of the semi-spans on either side. Referring to Fig. 1, the change is just the vertical component of tension at midspan,

$$V_M = H \cdot \frac{h}{S} \tag{15}$$

Thus, the vertical load at support i is,

$$V_i = \frac{w}{2} \cdot (S_i + S_{i+1}) + H \cdot \left(\frac{h_i}{S_i} - \frac{h_{i+1}}{S_{i+1}} \right) \tag{16}$$

The longitudinal spring constant for that support becomes,

$$K_i = \frac{V_i}{l} = \frac{1}{l} \cdot \left[\frac{w}{2} \cdot (S_i + S_{i+1}) + H \cdot \left(\frac{h_i}{S_i} - \frac{h_{i+1}}{S_{i+1}} \right) \right] \tag{17}$$

The proper values of H and w are those after the change in temperature and/or load.

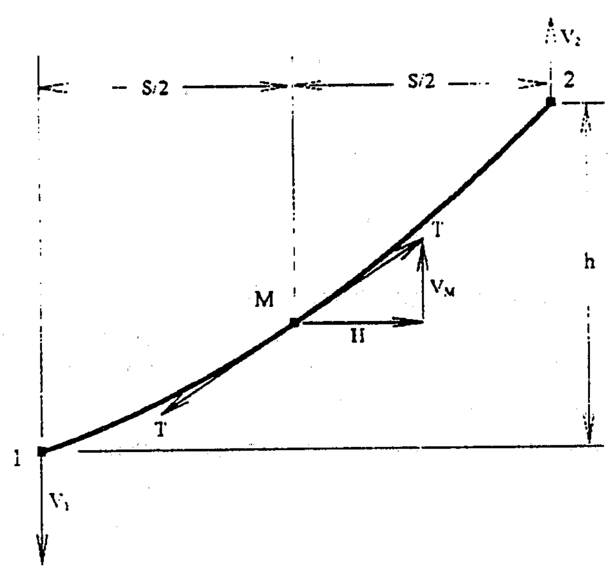

Figure 1

Nomenclature

- a_i Longitudinal movement of support to the right of span i for effectively infinitely long insulator string, due to change in temperature and/or loading.
- A Total conductor area.
- A_A Aluminum area.
- A_S Steel area.

E	Effective elastic modulus of composite conductor.
E_A	Young's Modulus for aluminum.
E_S	Young's Modulus for steel.
h_i	Difference in support point elevations for span i.
H	Horizontal component of conductor tension.
H_1	Value of H when insulators are plumb.
H_2	Value of H in ruling span after change in temperature and/or load.
H_i	Value of H in span i after change in temperature and/or/load.
k	Effective longitudinal spring constant of span
K_i	Effective longitudinal spring constant of tangent support to right of span i.
k_c	Elastic spring constant of conductor span, EA/S.
k_s	Effect spring constant of span due to variation of tension with sag.
l	Length of suspension insulator string.
S	Span length.
S_r	Ruling span.
S_w	Weight span.
t	Temperature.
T	Conductor tension.
V_i	Vertical load at support to right of span i.
w	Conductor weight per unit length.
x_i	Final longitudinal displacement of support to right of span i, after change in temperature and/or load.
α	Thermal coefficient of expansion of composite conductor.
ε_c	Elastic strain of conductor.
ε_p	Nonelastic (plastic or creep) strain of conductor.
ε_s	Difference between arc length and secant span length, expressed as strain.

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